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LEPTON MASS AND MIXING MATRICES INVOLVING A 17 keV DIRAC TAU NEUTRINO

Carl H. ALBRIGHT

Department of Physics, Northern Illinois University, DeKalb, Illinois 60115*

and

Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510†

Abstract

We extend our study of neutrino masses and mixings to the case of a 17 keV Dirac tau neutrino, which has reappeared in recent beta decay experiments. A special set of Dirac submatrices is inputed which works well for quarks and yields a top quark mass near 135 GeV. Unlike our previous Majorana neutrino study, however, we can not simultaneously explain all the data including the 17 keV neutrino, its small $|V_{\nu_\tau e}|^2 = 0.0085$ coupling to the electron, the present accelerator neutrino oscillation bounds, and the preferred solar neutrino nonadiabatic MSW effect interpretation.

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*Permanent address

†BITNET Address: ALBRIGHT@FNAL

The author[1] has recently determined neutrino masses and mixings from a special set of quark mass matrices used as input for the leptonic Dirac submatrices. These quark mass matrices, with just six real parameters, fit all the flavor-changing data remarkably well[2] with a top quark mass near 135 GeV. By varying the three Dirac neutrino parameters and the three diagonal Majorana submatrix entries, he was able to determine the allowed regions in the δm^2_{ij} vs. $\sin^2 2\theta_{ij}$ planes. Several interesting observations could be made, among them that the expected solar neutrino capture rate in gallium detectors should be less[3] than 25 - 40 SNU, and even more importantly, that the next-generation neutrino oscillation experiments should find some positive signals. This study was tacitly based on the "conventional" assumption that all neutrinos are Majorana in nature.

In the meantime, recent beta decay experiments[4] involving ^{14}C and ^{35}S appear to confirm the earlier findings of Simpson and Hime,[5] that the observed beta decay spectra for ^{3}H and ^{35}S exhibit an anomaly which can be due to a small admixture of a 17 keV neutrino. If this effect is indeed real, the 17 keV neutrino must be Dirac in character, for no signal for neutrinoless double beta decay necessarily involving a "heavy" Majorana neutrino has been found todate.[6] In this paper we repeat our previous analysis but now require the appearance of a 17 keV Dirac neutrino in the mass spectrum with the small observed[4] coupling probability, $\sin^2\theta \simeq 0.0085$. Glashow[7] has also recently considered this case and cited several characteristic features, including a discussion of the important role played by a scalar Majoron, if ν_τ is to be sufficiently short-lived. Here we shall concentrate on the masses and mixings in order to point out some apparent difficulties.

The neutrino and charged lepton mass matrices are taken to have the symmetric forms

$$M_N = \begin{pmatrix} 0 & \mathbf{M}_N \\ \mathbf{M}_N^T & \mathbf{M}_R \end{pmatrix}, \qquad M_L = \begin{pmatrix} 0 & \mathbf{M}_L \\ \mathbf{M}_L^T & 0 \end{pmatrix} \tag{1}$$

in the weak bases $\bar{B}_L = \{\bar{\nu}'_{iL},\ (\bar{\nu}'^c{}_i)_L\}$, $B_R = \{(\nu'^c_i)_R,\ \nu'_{iR}\}$ for the neutrinos and similarly for the charged leptons. We shall again identify the *forms* of the Dirac submatrices with the up and down quark mass matrices obtained previously according to

$$\mathbf{M}_N = \begin{pmatrix} 0 & A & A \\ A & A & B \\ A & B & C \end{pmatrix}, \qquad \mathbf{M}_L = \begin{pmatrix} 0 & iA' & -A' \\ -iA' & -A' & B' \\ -A' & B' & C' \end{pmatrix} \tag{2a}$$

and require the inequalities

$$\begin{aligned} |A| &\ll |B| \ll C \\ |A'| &\ll |B'| \ll C' \end{aligned} \tag{2b}$$

for a hierarchical chiral symmetry breaking pattern. With just six real parameters, these matrices for the up and down quarks, respectively, were found to fit the flavor-changing data exceedingly well,[2] while the Fritzsch[8] matrices fail rather badly with a top quark mass in excess of 100 GeV. For simplicity in presentation of the results, the Majorana submatrix $\mathbf{D}_M$ is taken to be diagonal and equal to

$$\mathbf{D}_M = diag(D_1,\ D_2,\ D_3) \tag{2c}$$

If all three families of neutrinos are pure Dirac, $\mathbf{D}_M$ is just the zero matrix. In this case, however, it is extremely difficult to understand why the neutrino parameters A_N, B_N, C_N should be many orders of magnitude smaller than their up quark counterparts. With two families of Dirac neutrinos, a pair of D_1, D_2, D_3 vanish, and the remaining Majorana neutrinos have the smallest and largest mass eigenvalues by the well-known seesaw mechanism.[9] The preferred neutrino spectrum suggested by the solar neutrino experiments is that both the electron and muon neutrinos are Majorana in character; hence we choose only the tau neutrino to be Dirac.

With the diagonal form (2c) for a $\mathbf{D}_M$ of rank 2, there are three locations where one can place the zero; with $D_1 = 0$, however, we find too small a mixing and can not vary it to obtain $|V_{\nu_\tau e}|^2 = 0.0085$, as suggested in Ref. 4. For most of the paper we shall set $D_2 = 0$ and later comment about the remaining choice, $D_3 = 0$, as well as other nondiagonal rank 2 forms. The 6×3 mixing matrix for the lefthanded lepton charged-current interactions is calculated by the projection operator trace technique of Jarlskog[10] as generalized by the author in Ref. 1, to which we refer the reader for details. For this purpose, the neutrino masses must be calculated to high precision from the eigenvalue equation for M_N.

The charged lepton mass matrix parameters A', B' and C' are determined uniquely by the known masses of the electron, muon and tau to be

$$A'_L = 0.007576, \qquad B'_L = 0.4181, \qquad C'_L = 1.686 \text{ GeV} \tag{3}$$

These differ from the down quark parameters by factors of just 0.3 - 0.6 as given in Ref. 1. The neutrino parameters A, B, C, D_1 and D_3 are determined as follows from a study of their dependence on the neutrino masses and mixing matrix elements.

(a) First set $B = 17.0$ keV which yields the apparently observed Dirac neutrino mass. Note that this is a factor of 10^{-6} smaller than its up quark matrix counterpart!

(b) Adjust $A = 1.54$ keV to obtain the observed coupling probability, $|V_{\nu_\tau e}|^2 \simeq 0.0085$. This is to be compared with $A = 75.5$ MeV for the up quark mass matrix entry.

(c) Consider values for C consistent with the hierarchy in (2b) and (3), which then determine the amount of $\nu'_e - \nu'_\mu$ mixing.

(d) The value of D_3 sets the scale for the ν_μ Majorana neutrino mass. Adjust D_3 for a given value of C, so that the ν_μ mass and $\sin^2\theta_{e\mu}$ lie along the allowed band[11]

$$\delta m^2_{e\mu} \sin^2\theta_{e\mu} = 10^{-8} \text{ eV}^2 \tag{4}$$

in the nonadiabatic Mikheyev-Smirnov-Wolfenstein[12] (MSW) solar neutrino region. The pairs of values for C and D_3 should lead to points in the $\delta m^2_{e\mu}$ vs $\sin^2 2\theta_{e\mu}$ plane presently allowed by the solar neutrino data.[13]

(e) Finally, D_1 sets the scale for the ν_e Majorana mass.

Unlike our previous study with six Majorana neutrinos, no arbitrary rescaling of all five parameters A, B, C, D_1 and D_3 is possible with one Dirac neutrino present.

We illustrate the form of the 6×3 mixing matrix which emerges for a special but typical and apparently acceptable case where

$$\begin{aligned} A &= 1.54 \text{ keV}, & B &= 17.0 \text{ keV}, & C &= 69.0 \text{ keV} \\ D_1 &= 550 \text{ GeV}, & D_2 &= 0, & D_3 &= 550 \text{ GeV} \end{aligned} \tag{4a}$$

This leads to

$$\begin{aligned} m_{\nu_e} &= 3.31 \times 10^{-7} \text{ eV}, & m_{\nu_\mu} &= 2.51 \times 10^{-4} \text{ eV} \\ m_{\nu_{\tau_1}} &= 17.1 \text{ keV}, & m_{\nu_{\tau_2}} &= 17.1 \text{ keV} \\ m_E &= 550 \text{ GeV}, & m_N &= 550 \text{ GeV} \end{aligned} \tag{4b}$$

with a mass difference for the two tau neutrino states of just 0.0089 eV. The squared mixing matrix elements are found to be

$$|V_{\alpha j}|^2 = \begin{pmatrix} 0.83873 & 0.16041 & 0.00086 \\ 0.15279 & 0.82022 & 0.02699 \\ 0.00424 & 0.00969 & 0.48608 \\ 0.00424 & 0.00969 & 0.48608 \\ 0.22 \times 10^{-19} & 0.41 \times 10^{-17} & 0.85 \times 10^{-20} \\ 0.11 \times 10^{-16} & 0.25 \times 10^{-17} & 0.44 \times 10^{-13} \end{pmatrix} \tag{4c}$$

where $\alpha = 1-6$ refers to ν_e, ν_μ, ν_{τ_1}, ν_{τ_2}, ν_E, ν_M and $j = 1-3$ refers to e, μ and τ. Note that all three columns of this matrix sum to unity; while rows 1 and 2 sum to unity, rows 3 and 4 sum only to 0.5. This is the hallmark of a Dirac neutrino, for only the lefthanded leptons couple in the standard model. Neglecting other exceeding small contributions, we write

$$\nu_\tau = \frac{1}{\sqrt{2}}(\nu_{\tau_1} + \nu_{\tau_2}), \qquad \nu_\tau^c = \frac{1}{\sqrt{2}}(\nu_{\tau_1} - \nu_{\tau_2}) \tag{4d}$$

where our choice of notation analogous to the neutral K meson system is deliberate. The effective 3×3 mixing matrix itself coupling the lefthanded leptons is then determined uniquely from the above in the standard sign convention to be

$$V = \begin{pmatrix} 0.916 & 0.401 & 0.029 \\ -0.391 & 0.906 & 0.164 \\ -0.092 & -0.139 & 0.986 \end{pmatrix} \tag{4e}$$

with a CP phase of $\delta = 0^\circ$. Here the rows refer sequentially to ν_e, ν_μ and ν_τ.

The neutrino flavor oscillations involving a Dirac tau neutrino which arise as a result of the time evolutions of the neutrino mass eigenstates are computed as follows. For a neutrino of flavor ν'_f produced at $t = 0$ in association with the lepton $f = e$, μ or τ, evolution of the weak flavor eigenstate into the following combination of mass eigenstates occurs after time t

$$|\nu'_f(t)> = V^*_{\nu_e f} e^{-i\omega_e t}|\nu_e> + V^*_{\nu_\mu f} e^{-i\omega_\mu t}|\nu_\mu> + \frac{1}{\sqrt{2}} V^*_{\nu_\tau f}\left(e^{-i\omega_{\tau_1} t}|\nu_{\tau_1}> + e^{-i\omega_{\tau_2} t}|\nu_{\tau_2}>\right) \tag{5a}$$

The probability that the weak eigenstate $|\nu'_f>$ has oscillated into $|\nu'_{f'}>$ after time t is then given by

$$\begin{aligned} \text{Prob}(\nu'_f \to \nu'_{f'}) &= |<\nu'_{f'}|\nu'_f(t)>|^2 \\ &= (1 - 2V_{\nu_\tau f}V_{\nu_\tau f'})\delta_{ff'} - 4V_{\nu_e f}V_{\nu_\mu f}V_{\nu_\mu f'}V_{\nu_e f'}\sin^2\tfrac{\omega_{\mu e}}{2}t \\ &\quad + V_{\nu_\tau f}\Big[V_{\nu_e f}V_{\nu_e f'}(\cos\omega_{\tau_1 e}t + \cos\omega_{\tau_2 e}t) + V_{\nu_\mu f}V_{\nu_\mu f'}(\cos\omega_{\tau_1\mu}t \\ &\quad + \cos\omega_{\tau_2\mu}t) + \tfrac{1}{2}V_{\nu_\tau f}V_{\nu_\tau f'}(3 + \cos\omega_{\tau_2\tau_1}t)\Big]V_{\nu_\tau f'} \end{aligned} \tag{5b}$$

Since $\omega_{ij}t \simeq \frac{m_i^2 - m_j^2}{2pc}L$, the term in $\sin^2\frac{\omega_{\mu e}}{2}t$ vanishes for all earth-based source-detector experiments and averages to 0.5 for solar neutrino experiments. The first four cosine terms in (5b) each average to zero for a 17 keV tau neutrino, since the oscillation length is typically less than 1 mm. The $\cos\omega_{\tau_2\tau_1}t$ term with $\delta m^2_{\tau_1\tau_2} = 304\ \mathrm{eV}^2$ in the example in (4) involves an oscillation length of 800 meters for 100 GeV neutrinos and must be taken into account in such accelerator experiments. In contrast, the two-component oscillation probabilities are determined by the experimentalists from the well-known formula

$$\begin{aligned}\mathrm{Prob}(\nu_f \to \nu'_{f'}) \ &= (1-2\delta_{ff'})\delta_{ff'} - 4V_{\nu_e f}V_{\nu_\mu f}V_{\nu_\mu f'}V_{\nu_e f'}\sin^2\tfrac{\omega_{\mu e}}{2}t \\ &\quad -4V_{\nu_e f}V_{\nu_\tau f}V_{\nu_\tau f'}V_{\nu_e f'}\sin^2\tfrac{\omega_{\tau e}}{2}t - 4V_{\nu_\mu f}V_{\nu_\tau f}V_{\nu_\tau f'}V_{\nu_\mu f'}\sin^2\tfrac{\omega_{\tau\mu}}{2}t\end{aligned} \tag{5c}$$

From the above, we draw the following interesting conclusions:

(1) While the mixing elements are quite small for the six Majorana neutrino case discussed in Ref. 1, in the presence of a Dirac ν_τ they are fairly large as shown in (4e). This example yields the oscillation parameters

$$\delta m^2_{e\mu} = 6.29\times 10^{-6}\ \mathrm{eV}^2, \qquad \sin^2 2\theta_{e\mu} = 0.519 \tag{6}$$

which locates the point at the nonadiabatic MSW - vacuum oscillation corner of the triangle in the $\delta m^2_{e\mu}$ vs $\sin^2 2\theta_{e\mu}$ plane. In fact, here the probability that ν'_e remains ν'_e over the sun-earth journey is calculated to be 74%.

(2) Difficulty is encountered, however, with the $\theta_{\mu\tau}$ oscillation angle derived from the above mixing matrix. Compared with the present experimental upper limits[14] for a 17 keV neutrino, we find the effective two-component oscillation parameters are given by

$$\sin^2 2\theta_{e\tau} = 0.025 \pm 0.008 < 0.12, \qquad \sin^2 2\theta_{\mu\tau} = 0.057 \pm 0.019 \not< 0.004 \tag{7}$$

The $\nu'_\mu \to \nu'_\tau$ mixing is clearly ruled out by Fermilab experiment E531.

(3) If we readjust the A parameter to satisfy the oscillation bound, the beta decay mixing is raised to $|V_{\nu_\tau e}|^2 \simeq 0.05$, too large by a factor of six for the most accurately quoted number in Ref. 4.

(4) In any case the mystery, already noted by Glashow,[7] remains why the parameters associated with the Dirac submatrices are orders of magnitude smaller than their quark and lepton counterparts. Here the rationale for the seesaw mechanism is lost, whereas in the six Majorana neutrino case one has the flexibility to make a tradeoff between the parameters in question and the Majorana masses which can be as large as $10^9 - 10^{12}$ GeV.

Can we do better with $D_3 = 0$? In this case $|V_{\nu_\tau e}|^2$ and $\sin^2 2\theta_{\mu\tau}$ are decoupled, being determined by both of the parameters A and B. The correct mixing results can be obtained, but only at the expense of $\delta m^2_{e\mu} \sin^2 \theta_{e\mu} \lesssim O(10^{-10}\mathrm{eV}^2)$ for $D_1 \sim D_2 \sim 300$ GeV, which is ruled out[13] by the solar neutrino data. The problem cited in (4) above is even worse. We have also studied several nondiagonal rank 2 forms for $\mathbf{M}_N$, but the same general results are obtained as discussed above.

We are unable to explain simultaneously the 17 keV neutrino, the small mixing $|V_{\nu_\tau e}|^2 = 0.0085$, the accelerator neutrino oscillation bounds, and the preferred solar neutrino nonadiabatic MSW effect interpretation within the model studied. By way of comparison, the Fritzsch model which is not favored by the quark data fares even worse. If all experiments are correct, it appears we must give up the simple connection between the quark and charged lepton mass matrices and the Dirac neutrino submatrices, or note that some additional contributions are present coming from some unknown singlet fermions, for example.

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